Collider signals from slow decays in supersymmetric models with an intermediate-scale solution to the μ problem

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Abstract

The problem of the origin of the μ parameter in the Minimal Supersymmetric Standard Model can be solved by introducing singlet supermultiplets with non-renormalizable couplings to the ordinary Higgs supermultiplets. The Peccei-Quinn symmetry is broken at a scale which is the geometric mean between the weak scale and the Planck scale, yielding a μ term of the right order of magnitude and an invisible axion. These models also predict one or more singlet fermions which have electroweak-scale masses and suppressed couplings to MSSM states. I consider the case that such a singlet fermion, containing the axino as an admixture, is the lightest supersymmetric particle. I work out the relevant couplings in several of the simplest models of this type, and compute the partial decay widths of the next-to-lightest supersymmetric particle involving leptons or jets. Although these decays will have an average proper decay length which is most likely much larger than a typical collider detector, they can occasionally occur within the detector, providing a striking signal. With a large sample of supersymmetric events, there will be an opportunity to observe these decays, and so gain direct information about physics at very high energy scales.

Contents

1 Introduction

Supersymmetry (for reviews, see [1, 2]) has the ability to stabilize the hierarchy between the electroweak and Planck scales. However, the minimal supersymmetric standard model (MSSM) still requires an explanation for the magnitude of the supersymmetric Higgs mass parameter μ. Assuming that there are no fine-tuned cancellations in the MSSM Higgs potential, μ should be of roughly the same magnitude as the soft supersymmetry-breaking masses. This suggests that μ arises as a vacuum expectation value (VEV) which is fixed by a potential with dimensionful parameters that are in turn determined by supersymmetry breaking.

Supersymmetry also requires some additional structure in order to solve the strong CP-problem. The μ parameter breaks the Peccei-Quinn (PQ) symmetry [3] that is otherwise naturally present in the MSSM at the renormalizable level, so it is an attractive proposition that the dynamics which leads to the μ term simultaneously provide for an invisible axion. Astrophysical constraints on the axion leave open a window [4] from roughly

$$10^9 \text{ GeV} \lesssim f \lesssim 10^{12} \text{ GeV} \tag{1.1}$$

for the VEV of the PQ-breaking field.

In this paper I will consider the phenomenology of a class of models with an invisible axion [5, 6] of the DFSZ type [6, 7, 8]. In these models, the μ term arises from non-renormalizable terms in the superpotential, for example:

$$W = \frac{\lambda_\mu}{M_P} X_1 X_2 H_u H_d. \tag{1.2}$$

(Here $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass, and λ_μ is a dimensionless coupling which I assume is not much smaller than unity.) The sum of the PQ charges of X_1 and X_2 must be equal and opposite to that of the MSSM Higgs superfields. When the scalar components of the neutral chiral superfields X_1 and X_2 acquire VEVs of order f, the approximate PQ symmetry is spontaneously broken, giving rise to a μ term of the right order of magnitude and an invisible axion. It is natural to assume that this occurs with X_1 and X_2 along a nearly flat direction in the potential. Then the low-energy degrees of freedom will typically include a pair of neutral chiral supermultiplets which are mixtures of the original X_1 and X_2. One of these contains as its imaginary scalar component the invisible axion of the model, and its fermion superpartner, the "axino". In some models, X_1 and X_2 are the same field, so that the axino is the only new light singlet fermion. If X_1 and X_2 are distinct, then there will be another light Majorana fermion "singlino". The axino and singlino both have odd R-parity and can mix. They obtain masses which are not much larger than the weak scale (but might be as small as of order a keV depending on the details of the model [9]). The upper limit can be understood from the facts that the (mass)^2 splittings between members of the same supermultiplet are bounded above by roughly $m_{3/2}^2$, the squared gravitino mass, and the axion is nearly massless.

The axino and the singlino are very weakly coupled to MSSM states, and cannot be directly produced in collider experiments at any significant rate. However, if either (or both) of these particles is lighter than all of the MSSM superpartners, then it will be the lightest supersymmetric particle (LSP) and can appear in decays from ordinary supersymmetric events.[†] In this paper I will argue there will be an opportunity in the era after supersymmetry is discovered to search for and measure the very long lifetime of the next-to-lightest supersymmetric particle (NLSP) into final states that include the singlino or axino, despite its very weak coupling.

In order to discuss the phenomenology in a general way, I will denote the relevant axino

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[†]If R-parity is conserved and the singlino or the axino is the LSP, it will be absolutely stable and could dominate the energy density of the universe too soon. This potential problem can be solved (as in many similar cases) by invoking a low reheat temperature, at the cost of requiring in addition a low-scale baryogenesis mechanism. Furthermore, the NLSP decays will safely occur long before nucleosynthesis in the standard cosmology.

or singlino LSP by $\widetilde{S}$, and refer to it generically as a singlino. It is part of a superfield S. In the low-energy theory it participates in a superpotential of the form

$$W = \mu \left(1 + \frac{\epsilon}{v}S\right) H_u H_d + \frac{1}{2} m_{\widetilde{S}} S^2 \tag{1.3}$$

in which

$$\mu = \frac{\lambda_\mu}{M_P} \langle X_1 \rangle \langle X_2 \rangle, \tag{1.4}$$

and I have introduced a dimensionless coupling parameter

$$\epsilon \sim v/f \tag{1.5}$$

with $v = 175$ GeV, the electroweak scale. (There are also soft mass terms for the scalar components of S, which will not concern us.) For example, if X_1 and X_2 are the same field, then one can read off from eq. (1.2) that $\epsilon = 2v/f$. For numerical purposes, this paper will use as a benchmark the value $\epsilon \approx 10^{-8}$ corresponding to $f \approx$ (few)$\times 10^{10}$ GeV. There is also a dimensionless, holomorphic soft term in the lagrangian

$$- \mathcal{L}_{\text{SUSY breaking}} = \frac{h_b}{M_P} X_1 X_2 H_u H_d + \text{c.c.} \tag{1.6}$$

where h_b is of order m_W. This gives rise to (among other terms) the necessary holomorphic soft (mass)^2 term for the Higgs bosons in the MSSM:

$$- \mathcal{L}_{\text{SUSY breaking}} = b H_u H_d + \text{c.c.} \tag{1.7}$$

Note that $b = h_b \langle X_1 \rangle \langle X_2 \rangle / M_P$ is of order m_W^2, as required for proper electroweak symmetry breaking.

The coupling ϵ and mass parameter $m_{\widetilde{S}}$ parameterize our ignorance of the high-energy theory. The smallness of ϵ means that S nearly decouples. However, the conservation of R-parity implies that if the singlino is the LSP, then decays of the NLSP to $\widetilde{S}$ will not suffer any competition and can be observed if they happen within a collider detector. These decays occur and are potentially observable because the "singlino" $\widetilde{S}$ mixes slightly with the gauginos and Higgsinos, as well as couples directly to higgsino-Higgs pairs. In that sense, these models are similar to the well-studied [10]-[15] "next-to-minimal supersymmetric standard model" (NMSSM) [16]. The differences include: the extremely small magnitude of ϵ; the fact that the field S does not obtain a VEV; the absence (or at least weak-scale phenomenological irrelevance) of an S^3 term in the superpotential; and the presence of a tree-level supersymmetric mass term for $\widetilde{S}$. Nevertheless, it is useful to compare the situation

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under study here to a very weakly coupled limit of the NMSSM. Indeed, the possibility of macroscopic decays involving a singlino have already been noted in ref. [15], but considering larger couplings (effectively $\epsilon \gtrsim 10^{-6}$) and for smaller values of $m_{\widetilde{S}}$ appropriate for LEP.

If the NLSP is the lightest of the ordinary MSSM neutralinos $\widetilde{N}_1$, then it can decay according to

$$\widetilde{N}_1 \to f\overline{f}\widetilde{S} \tag{1.8}$$

through virtual sleptons and squarks and virtual or on-shell Z bosons and Higgs bosons. The decay width is estimated very roughly by

$$\Gamma \sim \frac{m_{\widetilde{N}_1}}{16\pi} |\epsilon\mu/v|^2 \times \text{(suppression factors).} \tag{1.9}$$

The suppression factors include the effects of electroweak couplings, mixing angles, kinematic suppressions, and (if the mediating boson is not on-shell) three-body phase space. Without these effects, the rough estimate (for $m_{\widetilde{N}_1} \approx 100$ GeV and $|\epsilon\mu/v| \approx 10^{-8}$) would be of order 1 meter^{-1} for Γ.

After including the suppression effects in realistic models, we will find that when $\widetilde{N}_1$ is allowed to decay through an on-shell CP-even Higgs boson h^0, the inverse decay width is of order meters or tens of meters. Of course, the decay $\widetilde{N}_1 \to h^0\widetilde{S}$ may not be kinematically allowed. In that case, there may still be allowed decays $\widetilde{N}_1 \to Z^0\widetilde{S}$. These are typically further suppressed by a mixing angle, because the singlino must mix with the MSSM higgsinos in order to couple to the Z boson. Nevertheless, we will find that these mixing angles are typically large enough so that the inverse decay widths can be of order hundreds of meters. Finally, it may be that $m_{\widetilde{N}_1} - m_{\widetilde{S}} < m_Z$. In that case, there can still be three-body decays $\widetilde{N}_1 \to f\overline{f}\widetilde{S}$ through virtual sleptons, squarks, and the Z^0. (Decays through off-shell Higgs bosons can also occur, but are typically very small because the MSSM Higgs boson widths are tiny unless they are heavy.) If the Z boson is far off-shell, then with the usual model prejudices that sleptons are much lighter than squarks one finds that the smallest inverse decay widths are for $\ell^+\ell^-\widetilde{S}$ final states, and can be of order tens of kilometers. (All of these results assume $\epsilon \approx 10^{-8}$, and the decay widths must be scaled with ϵ^2.)

The majority of decays $\widetilde{N}_1 \to f\overline{f}\widetilde{S}$ will evidently occur well outside of a typical collider detector. However, with a significant number of supersymmetric events available, a small but finite fraction will occur inside the detector where the displaced secondary vertex can be distinguished. Since the decaying $\widetilde{N}_1$ and the resulting $\widetilde{S}$ are invisible, the experimental signature will involve an energetic lepton-antilepton pair or dijet pair with a significant opening angle appearing "from nothing" (with no corresponding charged particle track pointing back

to the interaction point) at a common point. This determination could be accomplished within the inner tracking volume of a detector, but might also be possible and perhaps even easier to distinguish if the decay occurs farther from the beam pipe. Thus, decays occurring within a meter to several meters from the interaction point could be a striking, if rare, signal.

At the Large Hadron Collider, the number of supersymmetric events expected per year with a low luminosity option of 10 fb^{-1}/year can be roughly of order a few thousand to a few million or more for 200 GeV $> m_{\widetilde{N}_1} > 50$ GeV. (This assumes $m_{\widetilde{g}} \approx m_{\widetilde{q}} \approx 7m_{\widetilde{N}_1}$; of course this is quite model-dependent.) Every event gives two possible NLSP decays. Therefore one can aspire to detect rare decays with widths as long as hundreds of kilometers by searching within the supersymmetric event sample [17]. In the limit of small decay widths, the probability that a particular $\widetilde{N}_1 \to f\overline{f}\widetilde{S}$ decay will occur within a distance L of the interaction point is given by

$$P(L) \approx L\Gamma/\beta\gamma \tag{1.10}$$

where Γ is the invariant partial width for that decay channel. Supersymmetric events will be "tagged" by the other particles from the sparticle decays and the presence of large $\not{E}_T$. Since the slowly decaying $\widetilde{N}_1$ will be massive and not ultrarelativistic, one can use timing information together with the pointing information from the tracking detectors and drift chambers and perhaps vetoes from the muon system to eradicate backgrounds from cosmic rays and other sources. There have also been proposals motivated by gauge-mediated supersymmetry breaking models and by neutralino decays to axino and photon [18] to build special detector components and instrumented tunnels to aid in the search for very slow decays [19, 20]. At future e^+e^- linear colliders, supersymmetric event rates are smaller, but the two-body decays mentioned above might occur often enough to be detected. The signal is somewhat more problematic at future runs of the Fermilab Tevatron collider, since the total supersymmetric event rates are likely to be considerably smaller. In this paper, I will simply remain optimistic and choose to present results for decay partial widths down to as small as $(1000 \text{ km})^{-1}$.

In these models, there are also singlet scalars S within the same supermultiplet as $\widetilde{S}$. These very weakly coupled scalars have masses of order m_W (or, in the case of the invisible axion, essentially 0). However, decays like $\widetilde{N}_1\widetilde{S}S$ depend on couplings that are effectively doubly suppressed by ϵ. Other decays involving only singlet scalars always have competition from ordinary unsuppressed MSSM decays, and so are not relevant for colliders.

The rest of this paper is organized as follows. In section 2, I examine some specific models which realize the idea outlined above. Section 3 discusses the couplings and mixings of the singlino/axino, and the relevant decays. Some representative numerical results for decays

to the singlino are presented in section 4. Section 5 contains some concluding remarks. An Appendix contains complete formulas for decay widths, including the effects of arbitrary phases.

2 Singlino masses and couplings in models with an intermediate-scale solution to the μ problem

Let us consider several models which realize the basic idea outlined in the introduction. The magnitude of $f \sim \langle X_{1,2} \rangle$ is (up to dimensionless couplings) the geometric mean of the Planck scale and the weak scale in order to agree with eq. (1.1). One way that this could happen is if the soft supersymmetry-breaking (mass)^2 of X_1 is driven negative at an intermediate scale. More generally, X_1 and X_2 correspond to a nearly-flat direction in the potential, so that dimensionless supersymmetry breaking terms involving X_1 and X_2 will always favor a non-trivial minimum at an intermediate scale. The key things we want to show are that these models generically contain one or more singlet fermions which have electroweak scale (or smaller) masses, and couplings that are of order v/f (with possible enhancements) to the MSSM Higgs fields.

For example, suppose that the superpotential contains, in addition to eq. (1.2), a term

$$W = \frac{\lambda_X}{6M_P} X_1 X_2^3, \tag{2.1}$$

as in [8]. This fixes the PQ charges of the superfields, ensuring the presence of an invisible axion provided that no other terms break the PQ symmetry. Here λ_X is a dimensionless coupling which is assumed to be of order unity. The supersymmetry breaking Lagrangian must then include

$$-\mathcal{L}_{\text{SUSY breaking}} = m_1^2 |X_1|^2 + m_2^2 |X_2|^2 - \frac{h_X}{6M_P} (X_1 X_2^3 + \text{c.c.}) \tag{2.2}$$

where h_X is a mass parameter of order the electroweak scale and has been taken to be real and positive without loss of generality. A nontrivial global minimum will exist provided e.g. that m_1^2 is negative at the scale of the VEV. However, it is important to note that this is not necessary. The presence of a holomorphic coupling h_X always favors spontaneous symmetry breaking at an intermediate scale. So VEVs for X_1 and X_2 can arise from a negative squared mass, and/or an h_X which is sufficiently large [21, 2]. In any case, $\langle X_1 \rangle$ and $\langle X_2 \rangle$ are of order $(m_W M_P)^{1/2} \sim 10^{10}$ GeV, which is naturally within the invisible axion window.

Let us parameterize the VEVs of X_1 and X_2 by an overall magnitude f and an angle ϕ, so that $\langle X_1 \rangle = f \cos \phi$ and $\langle X_2 \rangle = f \sin \phi$. Now expand the fields around their VEVs to obtain low-energy supermultiplet degrees of freedom S_1, S_2:

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix} f + \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}. \tag{2.3}$$

By requiring that the superpotential masses for the fermions $\widetilde{S}_1$ and $\widetilde{S}_2$ derived from eq. (2.1) are diagonal, one can solve for the mixing angle θ in terms of the VEV angle ϕ, with the result $\theta = \phi/2$. [This typically does not diagonalize the axion and other light scalar masses, and depends particularly on the choice of eq. (2.1).] The resulting masses and couplings for $\widetilde{S}_1$ and $\widetilde{S}_2$ are then found to be

$$m_{\widetilde{S}_1} = \frac{\lambda_X f^2}{2 M_P} \sin \phi \, (\cos \phi - 1); \qquad \epsilon_{\widetilde{S}_1} = \frac{v}{f} \left(\frac{\cos \phi/2}{\cos \phi} - \frac{\sin \phi/2}{\sin \phi} \right); \tag{2.4}$$

$$m_{\widetilde{S}_2} = \frac{\lambda_X f^2}{2 M_P} \sin \phi \, (\cos \phi + 1); \qquad \epsilon_{\widetilde{S}_2} = \frac{v}{f} \left(\frac{\sin \phi/2}{\cos \phi} + \frac{\cos \phi/2}{\sin \phi} \right). \tag{2.5}$$

[Compare eq. (1.3).] The scale f and the angle ϕ could also be computed, in principle, in terms of the parameters in the soft supersymmetry-breaking Lagrangian eq. (2.2) and the superpotential. The same parameters also determine the soft scalar mass of the saxion and other scalars with electroweak scale masses. However, these will not play any direct role in this paper, so I will not do this explicitly, and I will treat f and ϕ as free parameters.

One interesting limit is that of small ϕ (i.e., $\langle X_2 \rangle$ small compared to $\langle X_1 \rangle$), in which the mass eigenstate $\widetilde{S}_1$ is the axino. Then one finds that, up to phases,

$$m_{\widetilde{S}_1} = \frac{\lambda_X f^2}{4 M_P} \sin^3 \phi; \qquad \epsilon_{\widetilde{S}_1} = \frac{v}{2f}; \tag{2.6}$$

$$m_{\widetilde{S}_2} = \frac{\lambda_X f^2}{M_P} \sin \phi; \qquad \epsilon_{\widetilde{S}_2} = \frac{v}{f \sin \phi}. \tag{2.7}$$

Note that both masses become small in this limit. The coupling $\epsilon_{\widetilde{S}_1}$ must grow like $1/\sin \phi$ in this parameterization in order for μ to not become much less than v. (LEP has not discovered a higgsino.) Another interesting limit is $\phi = \pi/4$ (VEVs of equal magnitude), resulting in

$$m_{\widetilde{S}_1} = 0.10 \frac{\lambda_X f^2}{M_P}; \qquad \epsilon_{\widetilde{S}_1} = 0.77 v/f; \tag{2.8}$$

$$m_{\widetilde{S}_2} = 0.60 \frac{\lambda_X f^2}{M_P}; \qquad \epsilon_{\widetilde{S}_2} = 1.85 v/f, \tag{2.9}$$

again, up to phases. Both $\widetilde{S}_1$ and $\widetilde{S}_2$ have masses that are roughly of order the electroweak scale. In general they each contain an admixture of the axino. It is interesting to note that $|m_{\widetilde{S}_1}|/(\lambda_X f^2/M_P) < 0.1$ over the whole range $\theta < \pi/4$. So it is not unlikely that one or both of $\widetilde{S}_1$ and $\widetilde{S}_2$ is lighter than all MSSM sparticles.

Other models can be obtained by choosing superpotentials

$$W = \frac{\lambda_X}{6M_P}X_1 X_2^3 + \frac{\lambda_\mu}{2M_P}X_2^2 H_u H_d, \tag{2.10}$$

as in ref. [22], or

$$W = \frac{\lambda_X}{6M_P}X_1 X_2^3 + \frac{\lambda_\mu}{2M_P}X_1^2 H_u H_d. \tag{2.11}$$

In both of these cases, the diagonalized singlino masses are still given as in eqs. (2.4) and (2.5), since they only depend on the λ_X term in the superpotential. However, the couplings are modified to, respectively: and

$$\epsilon_{\widetilde{S}_1} = -\frac{2v \sin \phi/2}{f \sin \phi}; \qquad \epsilon_{\widetilde{S}_2} = \frac{2v \cos \phi/2}{f \sin \phi}. \tag{2.12}$$

for eq. (2.10), and

$$\epsilon_{\widetilde{S}_1} = \frac{2v \cos \phi/2}{f \cos \phi}; \qquad \epsilon_{\widetilde{S}_2} = \frac{2v \sin \phi/2}{f \cos \phi} \tag{2.13}$$

for eq. (2.11).

Another similar model is obtained by assuming a different form of the λ_X term used to stabilize the potential at large field strengths:

$$W = \frac{\lambda_X}{4M_P}X_1^2 X_2^2 + \frac{\lambda_\mu}{2M_P}X_1^2 H_u H_d. \tag{2.14}$$

Again using the mixing parameterization eq. (2.3), one finds in this class of models that now $\theta = (1/2)\tan^{-1}(2\tan 2\phi)$ in order that $\widetilde{S}_1$ and $\widetilde{S}_2$ are mass eigenstates. In terms of the VEV angle ϕ, we find:

$$m_{\widetilde{S}_1} = \frac{\lambda_X f^2}{4M_P}\left[1 - \cos 2\phi\sqrt{1 + 4\tan^2 2\phi}\right]; \tag{2.15}$$

$$\epsilon_{\widetilde{S}_1} = \frac{\sqrt{2}v}{f \cos \phi}\left[1 + \frac{1}{\sqrt{1 + 4\tan^2 2\phi}}\right]^{1/2}; \tag{2.16}$$

$$m_{\widetilde{S}_2} = \frac{\lambda_X f^2}{4M_P}\left[1 + \cos 2\phi\sqrt{1 + 4\tan^2 2\phi}\right]; \tag{2.17}$$

$$\epsilon_{\widetilde{S}_2} = \frac{\sqrt{2}v}{f \cos \phi}\left[1 - \frac{1}{\sqrt{1 + 4\tan^2 2\phi}}\right]^{1/2}, \tag{2.18}$$

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up to phases. In the limit $\phi \to 0$, one finds

$$m_{\widetilde{S}_1} = \frac{3\lambda_X f^2}{2M_P} \sin^2 \phi; \qquad \epsilon_{\widetilde{S}_1} = \frac{2v}{f}; \qquad (2.19)$$

$$m_{\widetilde{S}_2} = \frac{\lambda_X f^2}{2M_P}; \qquad \epsilon_{\widetilde{S}_2} = \frac{4v}{f} \sin \phi, \qquad (2.20)$$

which features a parametric suppression in the mass of $\widetilde{S}_1$. In the case of equal VEVs $\phi = \pi/4$, one obtains instead

$$m_{\widetilde{S}_1} = \frac{\lambda_X f^2}{4M_P}; \qquad \epsilon_{\widetilde{S}_1} = \frac{2v}{f}; \qquad (2.21)$$

$$m_{\widetilde{S}_2} = \frac{3\lambda_X f^2}{4M_P}; \qquad \epsilon_{\widetilde{S}_2} = \frac{2v}{f}. \qquad (2.22)$$

Finally, the limit $\phi \to \pi/2$ (i.e., $\langle X_1 \rangle$ small compared to $\langle X_2 \rangle$) yields a very light $\widetilde{S}_2$ and a relative enhancement of the $\widetilde{S}_1$ coupling:

$$m_{\widetilde{S}_1} = \frac{\lambda_X f^2}{2M_P}; \qquad \epsilon_{\widetilde{S}_1} = \frac{2v}{f \cos \phi}; \qquad (2.23)$$

$$m_{\widetilde{S}_2} = \frac{3\lambda_X f^2}{2M_P} \cos^2 \phi; \qquad \epsilon_{\widetilde{S}_2} = \frac{4v}{f}. \qquad (2.24)$$

There are clearly many possible more complicated variations on these models; for example, schemes with more than two fields X_i participating in the PQ-breaking and μ-generating dynamics (see, for example, ref. [23]). It is also possible to have a scheme in which there is only one field X, which obtains a VEV at an intermediate scale below where the soft mass term m_X^2 runs negative. This corresponds to the $\phi \to 0, \lambda_X \to 0$ limit of eq. (2.14) with $\widetilde{S}_2$ removed, so there is just a light axino with $\epsilon = 2v/f$. The essential features of all these models are that they contain one or more singlino fields, with couplings naively of order v/f but which can be significantly enhanced, and which can easily be lighter than all of the MSSM odd-R-parity sparticles.

3   Mixing of the singlino with MSSM neutralinos

As shown in the previous section, one or both singlino mass eigenstates $\widetilde{S}_1$ or $\widetilde{S}_2$ can be lighter than all MSSM sparticles. In this section, I will consider the relevant mixings and couplings of such a singlino to the MSSM states, and the ensuing decay partial widths. I will use $\widetilde{S}$ to refer generically to either $\widetilde{S}_1$ or $\widetilde{S}_2$.

The properties of the singlino are determined by the superpotential eq. (1.3). At tree level, there are singlino-higgsino-Higgs boson couplings. Other couplings, including singlino-fermion-sfermion and singlino-higgsino-Z boson, arise due to singlino-gaugino and singlino-higgsino mixing. In order to discover the couplings of the $\widetilde{S}$ to the physical MSSM states, one must diagonalize the the 5×5 neutralino mass matrix. In the $(\widetilde{S}, \widetilde{B}, \widetilde{W}^0, \widetilde{H}_d^0, \widetilde{H}_u^0)$ basis, it is given by:

$$M^{(5)} = \begin{pmatrix} m_{\widetilde{S}} & 0 & 0 & -\epsilon\mu s_\beta & -\epsilon\mu c_\beta \\ 0 & M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -\epsilon\mu s_\beta & -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ -\epsilon\mu c_\beta & s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix}, \qquad (3.1)$$

where s_β, c_β stand for $\sin\beta$, $\cos\beta$, and s_W, c_W for $\sin\theta_W$, $\cos\theta_W$. In the following, I shall take $m_{\widetilde{S}}$ to be real and positive without loss of generality. This allows ϵ to have an arbitrary phase. Now, the off-diagonal terms proportional to ϵ can be treated as a perturbation. Therefore, our procedure is to first diagonalize $M^{(4)}$, the lower right 4×4 mass sub-matrix. This is accomplished with a unitary matrix Z_{ij} $(i, j = 1, \ldots, 4)$ according to:

$$Z_{ik}^* M_{kl}^{(4)} Z_{jl}^* = \delta_{ij} m_{\widetilde{N}_j}. \qquad (3.2)$$

Here the masses $m_{\widetilde{N}_j}$ are real and positive; this can always be done, regardless of the relative complex phases of μ, M_1 and M_2. To the lowest order in a perturbative expansion in ϵ, the singlino $\widetilde{S}$ is a mass eigenstate, and the ordinary MSSM neutralinos have the same masses that they would have had if $\widetilde{S}$ were absent. I will choose an ordering scheme such that $\widetilde{S} = \widetilde{N}_0$, with $m_{\widetilde{S}} = m_{\widetilde{N}_0} < m_{\widetilde{N}_1} < m_{\widetilde{N}_3} < m_{\widetilde{N}_3} < m_{\widetilde{N}_4}$.

The full 5×5 neutralino-singlino mass matrix can then be diagonalized according to

$$N_{ik}^* M_{kl}^{(5)} N_{jl}^* = \delta_{ij} m_{\widetilde{N}_j}, \qquad (3.3)$$

where now $i, j = 0, 1, \ldots, 4$. To lowest order in a perturbation in ϵ, one finds that $N_{00} = 1$ and $N_{ij} = Z_{ij}$ for $i, j = 1, 2, 3, 4$. So one can write

$$N_{ij} = \begin{pmatrix} 1 & N_{0j} \\ N_{i0} & Z_{ij} \end{pmatrix}. \qquad (3.4)$$

The neutralino-singlino mixing elements can be determined in terms of the 4×4 Z_{ij}, the mass eigenvalues, and the parameter ϵ:

$$N_{0j} = \sum_{k=1}^{4} Z_{kj} \left[\epsilon^* \mu^* m_{\widetilde{N}_k} (s_\beta Z_{k3} + c_\beta Z_{k4}) + \epsilon\mu m_{\widetilde{S}} (s_\beta Z_{k3}^* + c_\beta Z_{k4}^*) \right] / (m_{\widetilde{N}_k}^2 - m_{\widetilde{S}}^2); \qquad (3.5)$$

$$N_{i0} = -\left[\epsilon\mu m_{\widetilde{N}_i} (s_\beta Z_{i3}^* + c_\beta Z_{i4}^*) + \epsilon^* \mu^* m_{\widetilde{S}} (s_\beta Z_{i3} + c_\beta Z_{i4}) \right] / (m_{\widetilde{N}_i}^2 - m_{\widetilde{S}}^2). \qquad (3.6)$$

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(Again, nothing has been assumed here about the complex phases of the parameters M_1, M_2, μ and ϵ.) This expansion is an excellent approximation because $|\epsilon|$ is very small. In particular, one can check by numerical diagonalization of eq. (3.1) that the perturbative result is very accurate unless the denominator $m^2_{\widetilde{N}_1} - m^2_{\widetilde{S}}$ in eqs. (3.5) and (3.6) is tuned to 0 to an extreme accuracy (comparable to $|\epsilon\mu m_{\widetilde{S}}|$), in which case the decay widths studied below are completely negligible anyway.

The relevant neutralino and singlino couplings for low energy phenomenology are all contained in the five eigenmasses $m_{\widetilde{N}_i}$ and $m_{\widetilde{S}}$ (for which corrections proportional to ϵ are negligible) and the neutralino mixing quantities N_{ij} and N_{0j} with $(i,j = 1,\ldots,4)$. The decay rates with a singlino in the final state can now be worked out, as in the NMSSM [12]. I present formulas for the relevant widths in an Appendix, taking care to allow for possible arbitrary phases in the neutralino mixing matrix and in ϵ. In general, the phase of ϵ could be anything, since it is not constrained by low-energy experiments on CP violation and is not necessarily correlated with the phase of μ. Because the results might be useful for other problems, I will provide the general results for a decay involving $\widetilde{N}_i$ to $\widetilde{N}_j$; the case needed in this paper is obtained by simply taking $i = 1$ and $j = 0$. Numerical illustrations of these results will be given in Section 4.

The necessary coupling for the decay

$$\widetilde{N}_1 \rightarrow h^0 \widetilde{S} \tag{3.7}$$

arises directly from the superpotential eq. (1.3) through the higgsino content of $\widetilde{N}_1$. It also obtains contributions from the singlino-higgsino mixings N_{03} and N_{04} combined with the gaugino content of $\widetilde{N}_1$, and from the singlino-gaugino mixing elements N_{01} and N_{02} combined with the higgsino content of $\widetilde{N}_1$. The total coupling is given explicitly by eq. (A.2). The Higgs will then decay according to $h^0 \rightarrow b\bar{b}$, WW^*, $\tau^+\tau^-$, $c\bar{c}$, or $g\bar{g}$, providing a visible product displaced from the original interaction point producing the event.

The decay

$$\widetilde{N}_1 \rightarrow Z\widetilde{S} \tag{3.8}$$

relies both on singlino-higgsino mixings N_{03} and N_{04} and on the higgsino content of $\widetilde{N}_1$, and is therefore somewhat more suppressed in models with a bino-like NLSP. The relevant coupling is given explicitly by eq. (A.6). The Z boson then decays with Standard Model branching fractions to quark-antiquark and lepton-antilepton pairs.

When neither two-body decay is open, the neutralino NLSP will decay through off-shell sleptons, squarks, the Z, and Higgs bosons. The full expressions for these decays are given

in the Appendix. The most important contribution for a bino-like NLSP is typically through sleptons, and therefore relies on the singlino-bino and singlino-wino mixing elements N_{01} and N_{02}. Fortunately, these are not greatly suppressed in many models unless $|\mu|$ is very large.

There remains the possibility of a two-body decay $\widetilde{N}_1 \to \gamma \widetilde{S}$, which arises at the one-loop level. However, using the results of [24] (with appropriate changes for the couplings to correspond to the model under present consideration), I have verified that these decays are always suppressed compared to the two- and three-body decays considered here, with widths that cannot be much larger than a few times $(1000 \text{ km})^{-1}$ in the examples in the next section with $\epsilon = 10^{-8}$. Since these decays are not competitive here, I will not present results for them.

A separate possibility is that the NLSP is a stau, or that all three lighter, mostly-right-handed, slepton mass eigenstates $(\widetilde{\tau}_1, \widetilde{\mu}_R, \widetilde{e}_R)$ have no open decays except to the singlino. In that case, one can hope to observe $\widetilde{\tau}_1 \to \tau \widetilde{S}$ (and perhaps $\widetilde{\mu}_R \to \mu \widetilde{S}$ and $\widetilde{e}_R \to e \widetilde{S}$). The decaying slepton will appear in the detector as a muon-like charged particle track, or as a track with an anomalously high ionization rate. The rare decay to the singlino will yield a large-angle kink in the track leading either to a tau jet or an electron or muon. Since the decaying particle is heavy, there will be a significant angle at the kink. The decay widths are suppressed only by the singlino-gaugino mixing, so they can occur within the detector often enough to measure, even if ϵ is significantly less than 10^{-8}.

4 Representative results for decays to the singlino

In this section, I will consider some illustrative numerical results for decays to the singlino, first for neutralino NLSP models and then for stau or slepton NLSP models. I will take $\epsilon = 10^{-8}$, with the understanding that the results have to be scaled according to $\Gamma \propto \epsilon^2$.

4.1 Neutralino decays

In order to study the decay partial widths of a neutralino, I will employ the concept of "model lines", in which one supersymmetry-breaking parameter is allowed to vary, setting the overall scale for all sparticle masses. First, consider a typical model scenario with a bino-like NLSP and the LSP singlino mass fixed at $m_{\widetilde{S}} = 50$ GeV. The bino mass parameter M_1 is varied, with the wino mass parameter M_2 and the μ term then determined according to $M_2 = 2.0 M_1$; $\mu = 3.0 M_1$. The 5×5 neutralino mass matrix is then fully determined by also choosing fixed values of $\tan\beta = 3.0$ and $\epsilon = 10^{-8}$. The right-handed slepton masses $m_{\widetilde{e}_R} = m_{\widetilde{\mu}_R} = m_{\widetilde{\tau}_R}$ are constrained to be the greater of $1.2 m_{\widetilde{N}_1}$ and 110 GeV. This assures
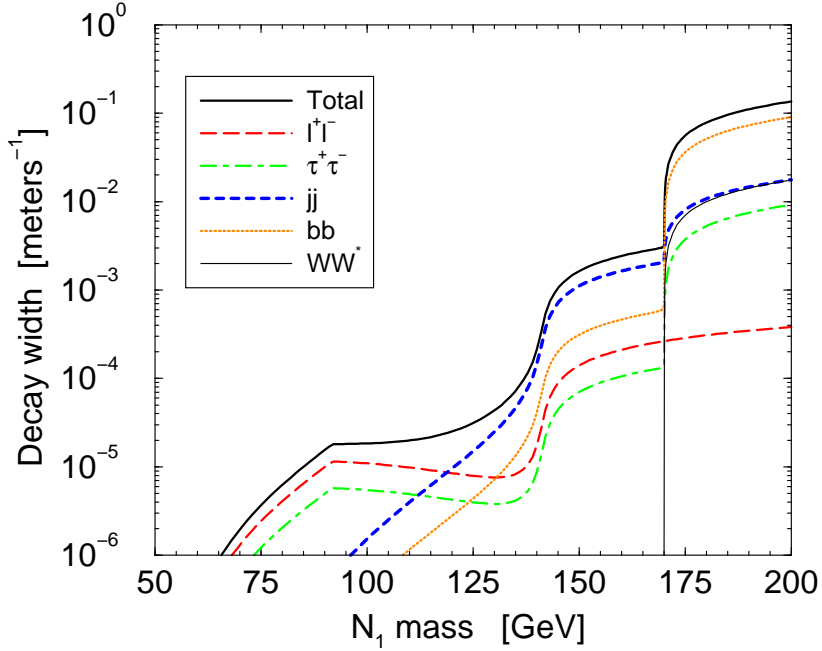
Figure 1: Visible decay widths of bino-like neutralinos $\widetilde{N}_1$ into final states involving the singlino, as a function of varying $m_{\widetilde{N}_1}$ for fixed $m_{\widetilde{S}} = 50$ GeV and $\epsilon = 10^{-8}$. The MSSM model parameters are described in the text. The solid line is the total visible decay width. The long dashed line is the partial width into $\ell^+\ell^-\widetilde{S}$ where $\ell = e$ or μ. The dot-dashed line is the width into $\tau^+\tau^-\widetilde{S}$. The short-dashed line includes $jj\widetilde{S}$ where j is any u,d,s,c (anti)-quark jet or gluon jet, and the dotted line is for $b\bar{b}\widetilde{S}$. The thin solid line is the width for $WW^*\widetilde{S}$ through an on-shell Higgs boson.

that a slepton cannot be the NLSP and should not be found at LEP. Mixing in the stau sector is neglected. The left-handed slepton masses are determined by $m_{\tilde{e}_L}^2 = m_{\tilde{e}_R}^2 + 0.5M_2^2$. I will assume that squarks are not light enough to give a significant contribution to the decay. Finally, the lightest Higgs boson mass is assumed to be $m_{h^0} = 120$ GeV, safely out of the reach of LEP, and to obey the decoupling limit $\alpha = \beta - \pi/2$. The results for the partial decay widths to visible states (excluding neutrinos), as found from the equations in the Appendix, are shown in Figure 1 as a function of $m_{\widetilde{N}_1}$.

For $m_{\widetilde{N}_1} \lesssim 120$ GeV in this model line, the decays are dominated by the contributions of the virtual right-handed sleptons. The total inverse decay lengths are of order tens of kilometers, and are nearly democratic between e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$ final states. The "knee" near $m_{\widetilde{N}_1} = 92$ GeV is merely an artifact of the constraint $m_{\tilde{e}_R} > 110$ GeV; for smaller neutralino masses, the virtual slepton is necessarily more off-shell due to the constraint that it has not been discovered at LEP.

For $m_{\widetilde{N}_1} \gtrsim 120$ GeV, the contributions from the virtual Z boson [the terms W_Z, W_{Zt}, and W_{Zu} in eq. (A.12)] begin to be important. This increases the decay partial widths, with contributions to $f\overline{f}$ that are roughly proportional to the Z branching fraction, so that dijet final states dominate. For $m_{\widetilde{N}_1} > 142$ GeV, the virtual Z boson is on-shell, and the decay becomes two-body $\widetilde{N}_1 \to Z^0 \widetilde{S}$. (The three-body formula is used in the vicinity of threshold, however, in order to correctly include interference effects with the virtual slepton diagrams in that regime.) This leads to a total visible decay width greater than $(1000 \text{ meters})^{-1}$. For negative μ, the decay widths tend to be somewhat smaller.

Finally, for $m_{\widetilde{N}_1} > m_{\widetilde{S}} + m_{h^0} = 170$ GeV in this model line, the decay $\widetilde{N}_1 \to h^0 \widetilde{S}$ opens up and completely dominates. Since there is a direct higgsino-singlino-Higgs boson coupling, this is much larger than the two-body decay to $Z\widetilde{S}$, even though the Z boson is lighter. The results are shown assuming Standard Model branching fractions for h^0 into final states $b\overline{b}$, WW^*, $\tau^+\tau^-$, and (lumped together into the "jj" category) $c\overline{c}$ and gg. Here the partial decay width of $\widetilde{N}_1$ to the $b\overline{b}\widetilde{S}$ final state is found to be of order $(10 \text{ meters})^{-1}$. Of course, if the h^0 mass is smaller, this mode will open up and dominate for smaller values of $m_{\widetilde{N}_1}$.

In the era after supersymmetry is discovered, the situation will be rather different; we will presumably know the MSSM sparticle mass spectrum, but the singlino mass and coupling will be completely unknown. So, a more useful summary of the situation we could face might be something like that shown in Figure 2. This is a particular point along the same model line just discussed, with $m_{\widetilde{N}_1} = 150$ GeV, and with the horizontal axis representing the possible values of the singlino mass.

A somewhat different scenario ensues if the NLSP is a higgsino-like neutralino. To illustrate this, I choose a pair of model lines with $\mu = \pm 0.8 M_1$, and all other parameter relationships as described above for Figure 1. The results are shown in Figure 3, but now only for the total visible decay width. Since $\widetilde{N}_1$ has a smaller gaugino content, the decays through virtual sleptons are highly suppressed. Conversely, the large higgsino component of $\widetilde{N}_1$ enhances the probability of decay through a virtual Z boson. So, for $m_{\widetilde{N}_1} < m_{\widetilde{S}} + m_{\widetilde{h}^0} = 170$ GeV in this model line, the $f\overline{f}$ decays will obey Z boson branching fractions. However, there turns out to be an accidental suppression of the matrix element for $\mu < 0$ in the convention specified in eq. (3.1) (which is the same as in refs. [1, 2, 10]), particularly when the Z boson is off-shell. For $m_{\widetilde{N}_1} > 170$ GeV, the decay $\widetilde{N}_1 \to h^0 \widetilde{S}$ length is of order several meters if $\widetilde{N}_1$ is mostly higgsino.

In the above analyses, I have assumed that H^0 and A^0 are very heavy, and that three-body amplitudes involving them are negligible. Although this is appropriate throughout most of parameter space, it is possible that for large $\tan\beta$, the couplings of H^0 and A^0 could
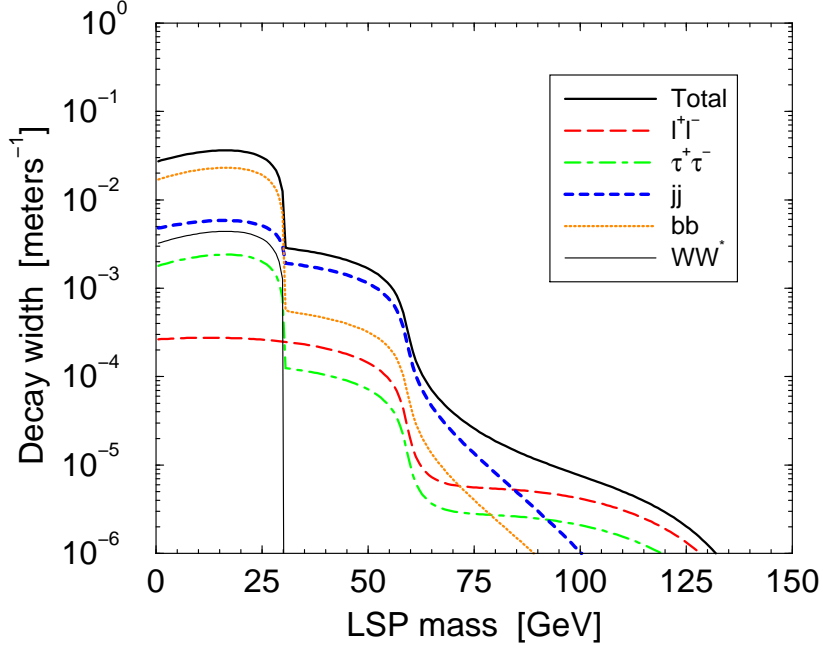
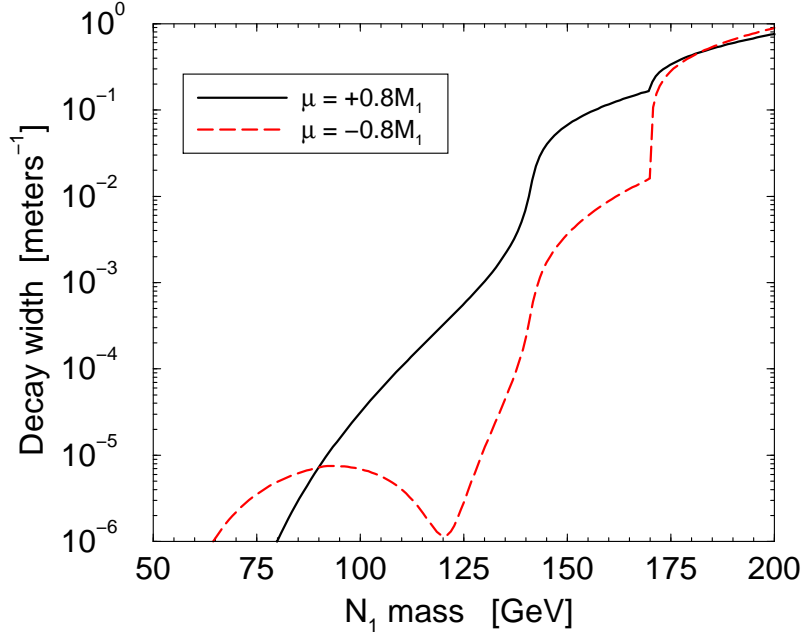Figure 2: As in Figure 1, but for fixed $m_{\widetilde{N}_1} = 150$ GeV, and varying $\widetilde{S}$ (LSP) mass.

Figure 3: Total visible decay widths of higgsino-like neutralinos $\widetilde{N}_1$ into final states $f\overline{f}\widetilde{S}$, as a function of varying $m_{\widetilde{N}_1}$ for fixed $m_{\widetilde{S}} = 50$ GeV. The model parameters satisfy the constraints $M_2 = 2.0M_1$, $\tan\beta = 3.0$, and $\mu = +0.8M_1$ (solid line) or $\mu = -0.8M_1$ (dashed line), and other constraints described in the text.

be large enough to make an appreciable contribution to $\Gamma(\widetilde{N}_1 \rightarrow b\overline{b}\widetilde{S})$, even if H^0 and A^0 are far off-shell.

Finally, I note that the decay widths discussed here depend on the phase of the parameter ϵ. This phase is not constrained by low-energy CP violating observables, and so might be considered completely arbitrary. I have checked, using the formulas in the Appendix, that varying $\text{Arg}(\epsilon)$, while keeping all other parameters fixed, can change the $\widetilde{N}_1$ decay widths by an order of magnitude or so. In models with a bino-like NLSP as in Figure 1, the largest decay widths tend to occur for real ϵ.

4.2   Slepton decays

It is also possible that the NLSP is a slepton $\widetilde{\tau}_1$ or, effectively, all three mainly right-handed sleptons $\widetilde{e}_R$, $\widetilde{\mu}_R$ and $\widetilde{\tau}_1$. The latter scenario is realized if $\tan\beta$ is not too large, so that the three sleptons are mass-degenerate to within less than m_τ. These possibilities are familiar [25]-[31] in gauge-mediated supersymmetry breaking models [32] but could also be realized in supergravity-mediated models if there is not a large universal contribution to scalar masses, or if D-term contributions (proportional to some exotic $U(1)$ quantum number) are large.

If $\widetilde{\tau}_1$ is the NLSP, then the two-body decays $\widetilde{\tau}_1 \rightarrow \tau\widetilde{S}$ are suppressed only by the bino-singlino mixing. In Figure 4, I show the results for this decay width for a typical model line with varying M_1 and fixed LSP mass $m_{\widetilde{S}}$. In order to ensure that a stau is the NLSP, the constraint $m_{\widetilde{\tau}_1} = 0.9m_{\widetilde{N}_1}$ (thick lines) and $0.7m_{\widetilde{N}_1}$ (thin lines) are imposed. Other relevant model line parameters are $M_2 = 2.0M_1$, $\mu = 3.0M_1$, $\tan\beta = 3.0$. To a good approximation, the decay width depends only on the absolute value of $s_{\widetilde{\tau}}$ in the stau mixing parameterization of eq. (A.24), so results are shown for $s_{\widetilde{\tau}} = 0$, 0.25, and 0.5. The solid line is also approximately true for $\widetilde{e}_R \rightarrow e\widetilde{S}$ and $\widetilde{\mu}_R \rightarrow \mu\widetilde{S}$ as a function of $m_{\widetilde{e}_R}$ and $m_{\widetilde{\mu}_R}$, by taking $s_{\widetilde{\tau}} = 0$, $c_{\widetilde{\tau}} = 1$.

Because there is relatively little suppression in this case, the inverse decay widths are of order tens of meters. This is discernible at a collider which can produce several hundred supersymmetric events. Each such event would contain a pair of quasi-stable stau or slepton highly ionizing tracks, which can have an anomalously high dE/dx to distinguish them from muons. In a small fraction of events, one of the stau or slepton tracks will have a kink leading to a lepton or tau jet, corresponding to the decay. The resulting tau or lepton would have a significant angle with respect to the original highly ionizing track, yielding a potentially spectacular and nearly background-free signal.
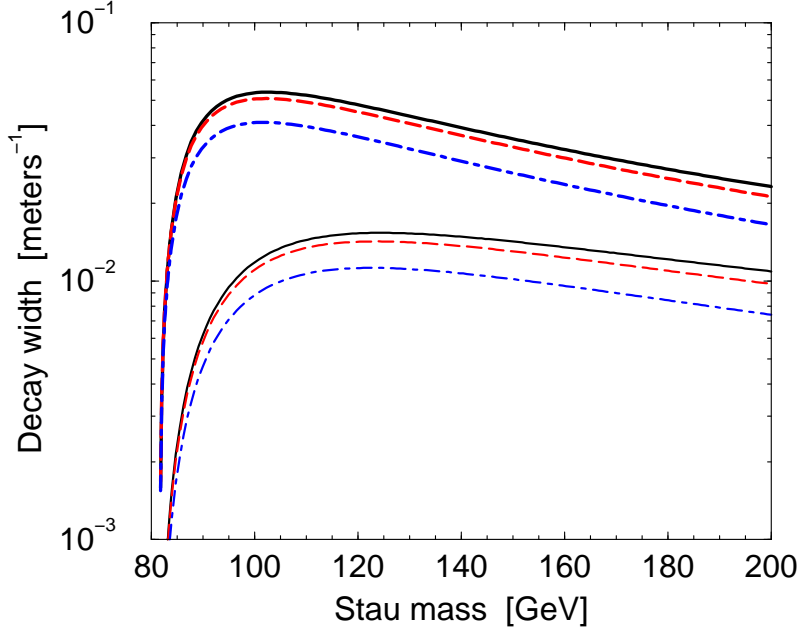
Figure 4: Decay widths for $\tilde{\tau}_1 \to \tau \tilde{S}$ as a function of varying $m_{\tilde{\tau}_1}$. The model parameters satisfy $m_{\tilde{S}} = 80$ GeV, and the constraints $M_2 = 2.0 M_1$, $\mu = 3.0 M_1$, and $\tan\beta = 3.0$. Solid lines are for the unmixed case $s_{\tilde{\tau}} = 0$, the dashed lines for $s_{\tilde{\tau}} = 0.25$ and dash-dotted lines for $s_{\tilde{\tau}} = 0.5$. The thicker (thinner) lines are for $m_{\tilde{\tau}_1}/m_{\tilde{N}_1} = 0.9$ (0.7).

5   Conclusions

The presence of the μ term in the MSSM and the solution to the strong CP problem may have a common explanation at an intermediate scale. Direct detection of the resulting axion is quite problematic. In this paper, I have argued that these models may nevertheless give rise to observable signals at colliders, through delayed decays to singlino fermions that include the axino as a mixture. These events will be a rare (perhaps very rare) occurrence within a large sample of supersymmetric events at the Large Hadron Collider or a future $e^+ e^-$ linear collider. There is also a possibility that the lightest MSSM sparticle could have slow decays into more than one singlino. Note that the LHC cross sections can be very large precisely when the NLSP decay widths are small.

The numerical estimate in section 4 of this paper have used $\epsilon = 10^{-8}$ for the singlino-higgsino-Higgs coupling parameter. Of course, the actual value could be significantly smaller. On the other hand, I showed that in some models the couplings are parametrically enhanced, and the mass of one or more singlinos is reduced, if one of the VEVs giving rise to the μ term is relatively small. Furthermore, the magnitude of ϵ can be significantly larger if the high

scale M_P is replaced by a somewhat lower scale that governs non-renormalizable operators, for example a string scale or a compactification scale that is not far above the apparent gauge coupling unification scale.

Future planning and analysis of collider physics experiments should take into account the possibility that the apparent LSP is actually unstable. Besides the models I have discussed here, there are at least two other plausible variations on the MSSM which can lead to delayed rare decays of what might appear, at first, to be the stable LSP.

First, gauge-mediated supersymmetry-breaking (GMSB) models [32] with a supersymmetry-breaking scale $\sqrt{F}$ that is not too large will give rise to decays that could have macroscopic proper lengths [25]-[31]. It is interesting to compare the reason for this to that in the models discussed in the present paper. In GMSB models, an estimate for a decay width of the NLSP to the goldstino/gravitino $\widetilde{G}$ is $\Gamma \sim m_W^5/16\pi(\sqrt{F})^4$, while in the decays to a singlino/axino LSP $\widetilde{S}$, the estimate is $\Gamma \sim m_W^3/16\pi f^2$. So NLSP decays are suppressed by the 4th power of the supersymmetry-breaking scale in GMSB, but only by the square of the PQ scale in light axino/singlino models. GMSB models can, in fact, give rise to signals which might be very difficult to distinguish from those discussed here. For example, if the NLSP is a neutralino with a significant higgsino content, it can have [25, 33, 26, 34] decays $\widetilde{N}_1 \to h^0\widetilde{G}$ and $\widetilde{N}_1 \to Z^0\widetilde{G}$ that look like the decays discussed in this paper. Or, if a stau is the NLSP, it can appear quasi-stable with rare decays $\widetilde{\tau}_1 \to \tau\widetilde{G}$ occurring within the detector. Second, one can have weak R-parity violating couplings [35] in the MSSM which could also give decays like $\widetilde{N}_1 \to \ell^+\ell'^-\nu$ or $\widetilde{N}_1 \to q\overline{q}'\nu$. These signatures could mimic those discussed in the present paper.

If these signals appear, it will be interesting to try to establish the correct explanation from among the competing hypotheses. The prize for doing so will be that we will gain an understanding of physics at scales far above those probed by direct sparticle production at colliders.

Appendix: Complete formulas for neutralino decay widths including the effects of arbitrary phases

In this appendix, I give formulas for the two- and three-body decays of a neutralino to another neutralino and a Higgs boson, Z boson, or fermion-antifermion pair. The model is

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the extension of the MSSM with one singlet superfield as specified by eq. (1.3). This also includes the MSSM and NMSSM as special cases. The results needed in Section 4 of this paper are obtained by taking the neutralino mass eigenstate indices to be $i = 1$ and $j = 0$ in the following.

First, let us consider the two-body decay of a neutralino to another neutralino and the lightest CP-even neutral Higgs boson. The relevant decay width is equal to:

$$\Gamma(\widetilde{N}_i \to h^0 \widetilde{N}_j) \quad = \quad \frac{m_{\widetilde{N}_i}}{16\pi} \sqrt{\lambda(1, r_j^2, r_{h^0}^2)} \Big(|G_{ij}^{h^0}|^2 (1 + r_j^2 - r_{h^0}^2) + 2\text{Re}[(G_{ij}^{h^0})^2] r_j \Big) \qquad \text{(A.1)}$$

where $r_j = m_{\widetilde{N}_j}/m_{\widetilde{N}_i}$; $r_{h^0} = m_{h^0}/m_{\widetilde{N}_i}$; $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$; and the neutralino-neutralino-Higgs coupling is given by

$$G_{ij}^{h^0} \quad = \quad \frac{1}{2}(gN_{i2}^* - g'N_{i1}^*)(s_\alpha N_{j3}^* + c_\alpha N_{j4}^*) + \frac{\epsilon\mu}{\sqrt{2}v}(c_\alpha N_{i3}^* - s_\alpha N_{i4}^*)N_{j0}^* + (i \leftrightarrow j). \quad \text{(A.2)}$$

Here c_α and s_α denote $\cos\alpha$ and $\sin\alpha$ with α the Higgs mixing angle in the notation of [10].

The results for the decays to the heavier neutral CP-even (H^0) and CP-odd (A^0) Higgs bosons can be obtained by instead using the couplings:

$$G_{ij}^{H^0} \quad = \quad \frac{1}{2}(gN_{i2}^* - g'N_{i1}^*)(-c_\alpha N_{j3}^* + s_\alpha N_{j4}^*) + \frac{\epsilon\mu}{\sqrt{2}v}(s_\alpha N_{i3}^* + c_\alpha N_{i4}^*)N_{j0}^* + (i \leftrightarrow j) \quad \text{(A.3)}$$

for $\widetilde{N}_i \to H^0 \widetilde{N}_j$, and

$$G_{ij}^{A^0} \quad = \quad \frac{i}{2}(gN_{i2}^* - g'N_{i1}^*)(s_\beta N_{j3}^* - c_\beta N_{j4}^*) + i\frac{\epsilon\mu}{\sqrt{2}v}(c_\beta N_{i3}^* + s_\beta N_{i4}^*)N_{j0}^* + (i \leftrightarrow j) \quad \text{(A.4)}$$

for $\widetilde{N}_i \to A^0 \widetilde{N}_j$, and substituting $m_{h^0} \to m_{H^0}$ or m_{A^0} in the obvious way. However, in the numerical analyses of Section 4, H^0 and A^0 are assumed to be heavy and decoupled, so these decays are neglected.

Similarly, the two-body decay of a neutralino to another neutralino and a Z boson has a width given by

$$\Gamma(\widetilde{N}_i \to Z^0 \widetilde{N}_j) = \frac{m_{\widetilde{N}_i}}{16\pi} \sqrt{\lambda(1, r_j^2, r_Z^2)} \Big(|G_{ij}^Z|^2 [1 + r_j^2 - 2r_Z^2 + (1 - r_j^2)^2/r_Z^2] + 6\text{Re}[(G_{ij}^Z)^2] r_j \Big) \qquad \text{(A.5)}$$

where $r_Z = m_Z/m_{\widetilde{N}_i}$, and the neutralino-neutralino-Z coupling is given by

$$G_{ij}^Z = \frac{g}{2c_W}(-N_{i3}N_{j3}^* + N_{i4}N_{j4}^*). \qquad \text{(A.6)}$$

Finally we consider three-body decays $\widetilde{N}_i \to f\overline{f}\widetilde{N}_j$, where f is any Standard Model quark or lepton. Results for these decays have appeared in refs. [12, 36, 37], but here I include

the effects of Higgs boson exchanges and arbitrary phases of the couplings . The differential partial widths can be expressed in terms of the dimensionless mass ratios $r_f = m_f/m_{\widetilde{N}_i}$, $r_j = m_{\widetilde{N}_j}/m_{\widetilde{N}_i}$, $r_Z = m_Z/m_{\widetilde{N}_i}$, $r_{\Gamma_Z} = \Gamma_Z/m_{\widetilde{N}_i}$, and $r_\phi = m_\phi/m_{\widetilde{N}_i}$ for each of $\phi = h^0, A^0$, and H^0. I use dimensionless kinematic variables

$$\hat{s} = (p_{\widetilde{N}_i} - p_{\widetilde{N}_j})^2/p_{\widetilde{N}_i}^2 \tag{A.7}$$

$$\hat{t} = (p_{\widetilde{N}_i} - p_f)^2/p_{\widetilde{N}_i}^2 \tag{A.8}$$

$$\hat{u} = 1 + r_j^2 + 2r_f^2 - \hat{s} - \hat{t} \tag{A.9}$$

with limits of integration

$$\hat{t}_{\min,\max} = \frac{1}{2}[1 + r_j^2 - \hat{s} + 2r_f^2 \mp \{(1 - 4r_f^2/\hat{s})\,\lambda(1, \hat{s}, r_j^2)\}^{1/2}]; \tag{A.10}$$

$$\hat{s}_{\min} = 4r_f^2; \qquad \hat{s}_{\max} = (1 - r_j)^2. \tag{A.11}$$

The results for widths can be expressed as[†]

$$d\Gamma(\widetilde{N}_i \to f\overline{f}\widetilde{N}_j) = \frac{n_c m_{\widetilde{N}_i}}{512\pi^3}\left(\sum W\right)d\hat{t}d\hat{s}, \tag{A.12}$$

where $n_c = 1$ (3) for leptons (quarks). The individual contributions to $\sum W$ are:

$$W_Z = \frac{4(a_f^2 + b_f^2)}{(\hat{s} - r_Z^2)^2 + r_Z^2 r_{\Gamma_Z}^2}\Big\{|G_{ij}^Z|^2\left[(1 - \hat{u})(\hat{u} - r_j^2) + (1 - \hat{t})(\hat{t} - r_j^2)\right]$$
$$+ 2\mathrm{Re}[(G_{ij}^Z)^2]r_j\hat{s}\Big\} \tag{A.13}$$

$$W_t = \sum_{n,n'=1}^{2}(a_j^n a_j^{n'*} + b_j^n b_j^{n'*})(a_i^{n*}a_i^{n'} + b_i^{n*}b_i^{n'})\frac{(1 - \hat{t})(\hat{t} - r_j^2)}{(r_{\tilde{f}_n}^2 - \hat{t})(r_{\tilde{f}_{n'}}^2 - \hat{t})} \tag{A.14}$$

$$W_u = W_t(\hat{t} \to \hat{u}) \tag{A.15}$$

$$W_{tu} = 2\mathrm{Re}\sum_{n,n'=1}^{2}\frac{1}{(r_{\tilde{f}_n}^2 - \hat{t})(r_{\tilde{f}_{n'}}^2 - \hat{u})}\Big[(a_j^n b_j^{n'}a_i^{n'*}b_i^{n*} + a_j^{n'}b_j^n a_i^{n*}b_i^{n'*})(r_j^2 - \hat{t}\hat{u})$$
$$+ (a_j^n a_j^{n'}a_i^{n*}a_i^{n'*} + b_j^n b_j^{n'}b_i^{n*}b_i^{n'*})\hat{s}r_j\Big] \tag{A.16}$$

$$W_{Zt} = \frac{4(\hat{s} - r_Z^2)}{(\hat{s} - r_Z^2)^2 + r_Z^2 r_{\Gamma_Z}^2}\mathrm{Re}\sum_{n=1}^{2}\Big[(a_f a_i^n a_j^{n*} + b_f b_i^{n*}b_j^n)\{G_{ij}^Z(1 - \hat{t})(\hat{t} - r_j^2)$$
$$+ G_{ij}^{Z*}\hat{s}r_j\}\Big]/(r_{\tilde{f}_n}^2 - \hat{t}) \tag{A.17}$$

$$W_{Zu} = W_{Zt}(\hat{t} \to \hat{u}) \tag{A.18}$$

$$W_{h^0,H^0} = \sum_{\phi,\phi'=h^0,H^0}\frac{4\hat{s}\,\mathrm{Re}[G_f^\phi G_f^{\phi'*}]}{(r_\phi^2 - \hat{s})(r_{\phi'}^2 - \hat{s})}\left\{(1 + r_j^2 - \hat{s})\mathrm{Re}[G_{ij}^\phi G_{ij}^{\phi'*}] + 2\hat{s}r_j\mathrm{Re}[G_{ij}^\phi G_{ij}^{\phi'}]\right\} \tag{A.19}$$

[†]In computing these results, I have neglected fermion masses arising from spinor algebra in matrix elements, but not in the kinematic limits of integration or the couplings.

$$W_{A^0} = \frac{4\hat{s}|G_f^{A^0}|^2}{(r_{A^0}^2 - \hat{s})^2}\left\{(1 + r_j^2 - \hat{s})|G_{ij}^{A_0}|^2 + 2\hat{s}r_j\text{Re}[(G_{ij}^{A_0})^2]\right\} \tag{A.20}$$

$$W_{\phi t} = -\sum_{\phi = h^0, H^0, A^0}\sum_{n=1,2}\frac{2}{(r_\phi^2 - \hat{s})(r_{\tilde{f}_n}^2 - \hat{t})}\text{Re}[\hat{s}G_f^\phi(\hat{t}G_{ij}^\phi + r_jG_{ij}^{\phi *})(a_i^{n*}b_j^n + a_j^{n*}b_i^n)] \tag{A.21}$$

$$W_{\phi u} = W_{\phi t}(\hat{t} \to \hat{u}). \tag{A.22}$$

Quantities appearing in the above results are as follows. First,

$$a_f = -\frac{g}{c_W}(T_{3f} - q_f s_W^2); \qquad b_f = -q_f g s_W^2/c_W \tag{A.23}$$

are the Z boson couplings to quarks and leptons with $(T_{3f}, q_f) = (1/2, 2/3)$ for up-type quarks, $(-1/2, -1/3)$ for down-type quarks, and $(-1/2, -1)$ for charged leptons. Left-right mixing and CP violation in the sfermion sector are parameterized by a unitary matrix, which I choose[‡] to write as

$$\begin{pmatrix} \tilde{f}_R \\ \tilde{f}_L \end{pmatrix} = \begin{pmatrix} c_{\tilde{f}} & s_{\tilde{f}} \\ -s_{\tilde{f}}^* & c_{\tilde{f}}^* \end{pmatrix}\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}, \tag{A.24}$$

where $|c_{\tilde{f}}|^2 + |s_{\tilde{f}}|^2 = 1$, and $m_{\tilde{f}_1} < m_{\tilde{f}_2}$, The resulting couplings for down-type sfermions $(\tilde{b}, \tilde{\tau})$ are:

$$a_i^{\tilde{f}_1} = \sqrt{2}s_{\tilde{f}}[gT_{3f}N_{i2}^* + g'(q_f - T_{3f})N_{i1}^*] - c_{\tilde{f}}^* gN_{i3}^* m_f/\sqrt{2}c_\beta m_W, \tag{A.25}$$

$$b_i^{\tilde{f}_1} = \sqrt{2}c_{\tilde{f}}^* g' q_f N_{i1} + s_{\tilde{f}} gN_{i3}m_f/\sqrt{2}c_\beta m_W \tag{A.26}$$

$$a_i^{\tilde{f}_2} = -\sqrt{2}c_{\tilde{f}}[gT_{3f}N_{i2}^* + g'(q_f - T_{3f})N_{i1}^*] - s_{\tilde{f}}^* gN_{i3}^* m_f/\sqrt{2}c_\beta m_W, \tag{A.27}$$

$$b_i^{\tilde{f}_2} = \sqrt{2}s_{\tilde{f}}^* g' q_f N_{i1} - c_{\tilde{f}} gN_{i3}m_f/\sqrt{2}c_\beta m_W \tag{A.28}$$

for $i = 0, 1, \ldots 4$. For up-type fermions one must replace $N_{i3}^{(*)}/c_\beta$ by $N_{i4}^{(*)}/s_\beta$ in eqs. (A.25)-(A.28). (However, for the cases of interest in this paper, $\widetilde{N}_1 \to t\bar{t}\widetilde{S}$ is surely not kinematically allowed, and decays $\widetilde{N}_1 \to \nu\bar{\nu}S$ are not interesting.) In the expressions for the contributions to the widths, I have used the abbreviations $a_i^n = a_i^{\tilde{f}_n}$, etc. The various Higgs boson couplings to Standard Model fermions are given by, e.g.:

$$G_b^{h^0} = \frac{gm_b s_\alpha}{2m_W c_\beta}; \qquad G_t^{h^0} = -\frac{gm_t c_\alpha}{2m_W s_\beta}; \tag{A.29}$$

$$G_b^{H^0} = -\frac{gm_b c_\alpha}{2m_W c_\beta}; \qquad G_t^{H^0} = -\frac{gm_t s_\alpha}{2m_W s_\beta}; \tag{A.30}$$

$$G_b^{A^0} = -i\frac{gm_b\tan\beta}{2m_W}; \qquad G_t^{A^0} = -i\frac{gm_t\cot\beta}{2m_W}, \tag{A.31}$$

[‡]This parameterization has the feature that in the typical unmixed, CP-conserving case $c_{\tilde{f}} = 1$, $s_{\tilde{f}} = 0$, $\tilde{f}_1 = \tilde{f}_R$ and $\tilde{f}_2 = \tilde{f}_L$, with no minus signs.

for bottom and top quarks. The Higgs couplings for taus are obtained by $m_b \to m_\tau$, and for other quarks and leptons by the obvious substitutions.

The two-body decay width for a stau to a neutralino or singlino is given by:

$$\Gamma(\widetilde{\tau}_1 \to \tau \widetilde{N}_i) = \frac{m_{\widetilde{\tau}_1}}{16\pi}\sqrt{\lambda(1, r_i^2, r_\tau^2)}\Big\{(|a_i^{\widetilde{\tau}_1}|^2 + |b_i^{\widetilde{\tau}_1}|^2)(1 - r_i^2 - r_\tau^2) - 4r_\tau r_i \mathrm{Re}[a_i^{\widetilde{\tau}_1} b_i^{\widetilde{\tau}_1 *}]\Big\} \quad \text{(A.32)}$$

where now $r_i = m_{\widetilde{N}_i}/m_{\widetilde{\tau}_1}$ and $r_\tau = m_\tau/m_{\widetilde{\tau}_1}$, and the couplings $a_i^{\widetilde{\tau}_1}$, $b_i^{\widetilde{\tau}_1}$ are given already by eqs. (A.25)-(A.28). In section 4, this formula is used with $i = 0$, corresponding to $\widetilde{\tau}_1 \to \tau \widetilde{S}$. The results for $\widetilde{\mu}_R \to \mu \widetilde{S}$ and $\widetilde{e}_R \to e \widetilde{S}$ are obtained by taking $c_{\widetilde{\tau}} \to 1$ and $s_{\widetilde{\tau}} \to 0$.

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